BULL. P.M.E. (T.I.T.), *No. 48, September 1981* 621.375.826

The Real-time Measurement System for Stability of Laser Frequency*

By

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(Received May 26, 1981)

The authors have made an Allan variance real-time processing system (ARPS) to measure the laser frequency stability, where the Allan variance is a measure of frequency stability, i.e., the average of standard deviations of samples of two successive averaged frequencies. The ARPS has two frequency counters of 24 bit length and an Allan variance processor using a micro processor LSI 6802. The frequency counters are also controlled by the micro processor, and have zero dead time of the gate. The ARPS unables us to compute the Allan variance in wide range of integration time τ, that is, $1\ \mathrm{ms} \leq \tau < 2^{32} \times f^{-1}$ where f is the frequency of the input signal measured in Hz. From this system we can obtain Allan variances on two different signals independently.

1. Introduction

There have been considerable works on the stabilization of laser frequency,[1-5] and the stability has been improved up to 10^{-13}, which is enough to be utilized as a frequency standard. Some of the authors (M. O. and T. T.) have developed the frequency-offset-locked He-Xe laser system at 3.51μm.[5-7] The frequency stability of all three lasers in this system have to be simultaneously measured at real time when this system operates. For measuring the laser frequency stability, a frequency counter with an off-line mini computer or a real-time computing counter (HP5390A) is conventionally used. These are not necessarily suitable for measuring the frequency stability of many lasers simultaneously, since they are designed as universal measurement systems and moreover complex and expensive.

The square root of the Allan variance σ_y^2[8-10] is used as a typical measure of frequency stability, which is the average of standard deviations of samples of two successive averaged frequencies. Since only a simple arithmetical operation is required for the computation, it is suitable to apply a micro processor to a single purpose system for calculating the Allan variance.

For the purpose of computing and characterizing the frequency stability of a group of lasers the authors have developed a real-time system for measuring the stability of laser frequency, which we call an Allan variance real-time processing system, ARPS, for short.

2. The Measurement of Laser Frequency Stability

The Allan variance is defined as

$$\sigma_y^2(\tau) = \frac{1}{n-1} \sum_{k=1}^{n-1} \frac{(\bar{y}_{k+1} - \bar{y}_k)^2}{2}, \qquad (1)$$

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* Originally published in Trans. IECE Vol. J64-C No. 3 (1981)

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Published by Research Laboratory of Precision Machinery and Electronics,
Tokyo Institute of Technology, Nagatsuta, Midori-ku, Yokohama 227, Japan

where we define,

$$\bar{y}_k = \bar{f}_k / \bar{f},$$

$\bar{f}_k$: the k-th averaged signal frequency measured within the integration time τ.
$\bar{f}$: the nominal frequency of the signal,
τ : an integration time, and
n : the number of the data $\bar{f}_k$'s.

In Eq. (1), $\bar{f}_k$ and $\bar{f}_{k+1}$ are successively measured with zero dead time of the gate. This situation is schematically explained in Fig. 1.

Figure 2 shows the block diagram for the measurement of laser frequency stability using the beat signal between two lasers.[7] The frequency stability of each laser is estimated by calculating the Allan variance using counts of the beat frequency.

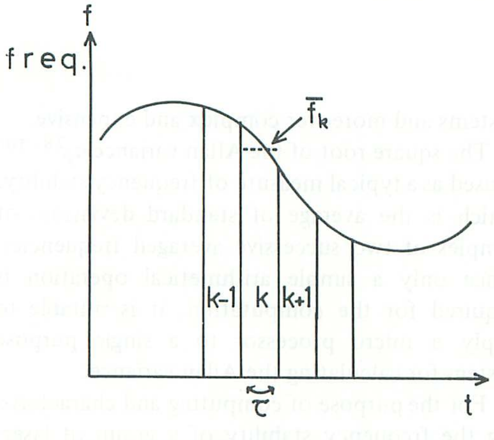

Fig. 1. Frequency change of the signal source.

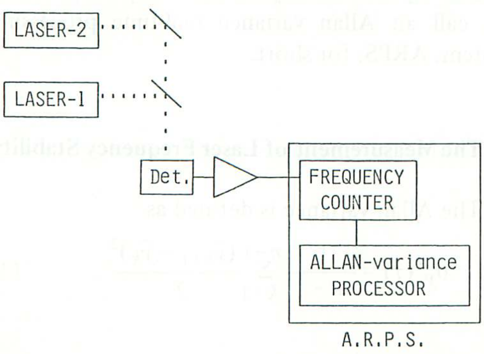

Fig. 2. The measurement system of the laser frequency stability.

3. The Hardware of the ARPS

Figure 3 shows the block diagram of the ARPS. An 8 bit micro processing unit (MPU) LSI 6802 is used.[11] All the programs including floating-point operations routines are fixed on a read only memory (ROM), and the frequency data are stored in a read write memory (RWM) of 3 k byte. Each peripheral interface adapter (PIA) is an LSI processing two group of 8 bit and four 1 bit input and output ports.[11] They are available for data transfer and control of interrupt-request. One of the two PIA's is used for data transfer from keyboard switches and transfer to silent thermal printer as shown in Fig. 3. Another PIA controls interrupt-requests from counters which indicate that the gate time for count is over. The result of count are directly transferred to the data bus lines of the system using tri-state gates.

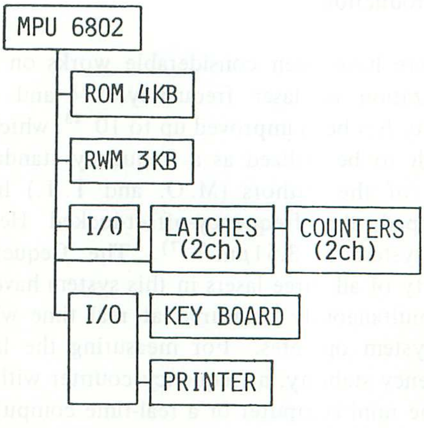

Fig. 3. The block diagram of the ARPS.

The frequency counters consist of two binary counters and latches (Fig. 4 (a)). As these counters work without reset at every gate time, they always overflow counts. The count is latched after the gate time. It takes several tens ns for counters to settle their counts after the rising edge of input signal. In order to prevent an ambiguity error, falling edge of the gate clock which decides a latch timing is delayed untill the

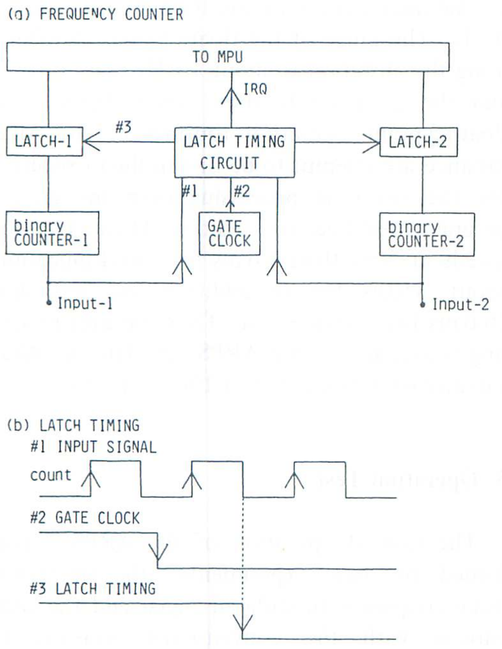

Fig. 4. The frequency counters.

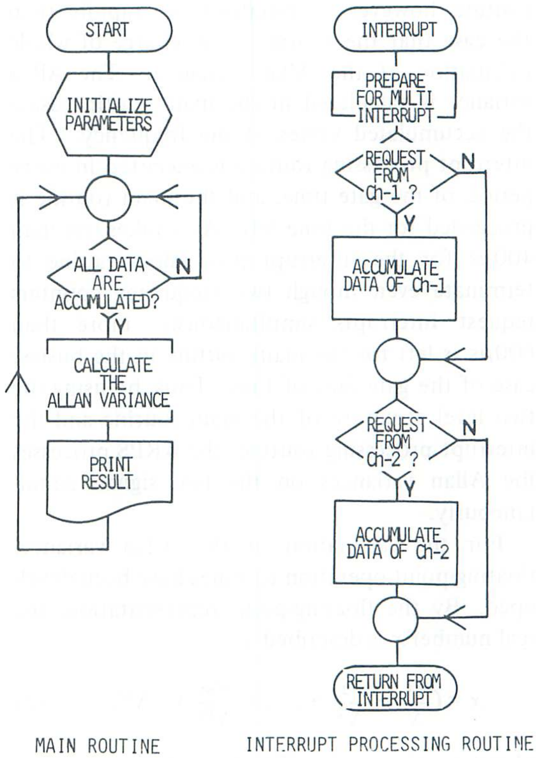

Fig. 5. The flowchart of the program.

falling edge of input signal. (Fig. 4 (b)) When the count is latched, the PIA switches the level of the interrupt-request (IRQ) bus line. Then the interrupt processing program runs where the acquisition of the data of count and calculation of the frequency are executed. As binary counters are 24 bit length, the accuracy of the frequency calculation is about 7 digit. Thus, the frequency is measured with zero dead time because counters are never reset.

4. The Software of the ARPS

Figure 5 shows the flowchart of the program. In the main routine, the parameters of the Allan variance are set initially by the operator. The n and τ are given to two measured values of counters, independently. The following instruction of the main routine makes a loop where a termination of acquisitions of appointed number of frequency data is checked. When the appointed number of frequency data n on one of the two input signals are acquired, data acquisitions on the input signal is stopped and the Allan variance is calculated. It takes about 0.7 s to complete calculation the Allan variance if the number of the data is 100.

On the other hand, as an interrupt-request occurs in every period of the gate time, the interrupt is identified and the data of count are acquired. As the interrupt-request occurs every period of the gate time of two frequency counters independently, the interrupt-request can occur when the interrupt processing routine is being executed. Considering this case, the ARPS is designed to be able to execute multi-level interrupt processing. In the interrupt processing routine, difference between two successive data of count is calculated and the difference is accumulated for the integration time τ. As the interrupt processing routine is used only for acquisition and accumulation of the data of count, the ARPS needs memory of the data number n for storage. The interrupt processing routine, however, terminates more rapidly than the case that the routine takes charge of whole calculation of the Allan variance. The Allan variance is calculated in the main routine using the accumulated values of the frequency. The interrupt processing routine is executed in every period of the gate time, and the main routine is proceeded for the time left. As it takes less than 400μs for the interrupt processing routine to terminate even though two frequency counters request interrupts simultaneously, more than 600μs is left for the main routine in the busiest case of the gate time of 1 ms. Thus, by using the two level structure of the main routine and the interrupt processing routine, the ARPS processes the Allan variances on the two signals simultaneously.

For the calculation of the Allan variance, floating-point operation routines have been developed. By the floating-point representation, the real number x is described as

$$x = (\frac{a_1}{N} + \frac{a_2}{N^2} + \ldots + \frac{a_m}{N^m}) \times N^{\alpha}, \qquad (2)$$

where $a_1, a_2, \ldots$ is a fixed point part, α is an exponent and N is a base. For the larger base N, the indicating range becomes wider, however the error increases due to the falling out of the fixed point part by some operations,[12] here 2 is adopted as N. The ARPS requires a high speed floating-point operation for real-time processing, and requires an accuracy of 3 byte length because subtractions between two frequency data of 3 byte length counters are executed in the calculation of the Allan variance. The data type of floating point data is shown in Fig. 6.[13] A floating-point number is expressed using 4 byte length which consists of a fixed point part of 3 byte, an exponent of 6 bit and signs of 1 bit

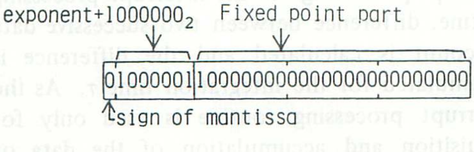

Fig. 6. The data type of floating-point data (the case of +1.).

for the fixed point part and the exponent, respectively. The range of the floating-point number x using this notation is about $10^{-19} < |x| < 10^{+19}$, and the accuracy is about seven digits. The floating-point operation routines of the Allan variance are attempt to be first in their execution for the real-time processing even though the program itself becomes tedious. Their execution speeds are less than 2.6 ms for a multiplication, about 350μs for an addition and less than 16.6 ms for a square root. Thus real-time processing is available as the ARPS calculate the Allan variance of data number of 100 within 1 s.

5. Operation Test

The normal operation of the ARPS is confirmed by two experiments – the simulation using frequency modulated signal and the comparison with the conventional measurement system.

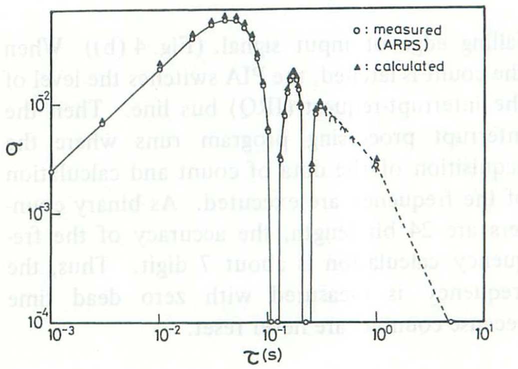

Fig. 7. The result of the operation test of the ARPS.

Figure 7 shows the result of simulation using frequency modulated signal, of which center frequency, the modulation frequency, and the maximum frequency deviation are 500 kHz, 8.6 Hz and 92 kHz, respectively. The Allan variance of the signal is calculated according to the Eq. (1), then

$$\sigma_y(\tau) = \frac{92}{500} \times \frac{\sin^2(\omega\tau/2)}{\omega\tau}, \qquad (3)$$

where $\omega = 2\pi \times 8.6$ rad/s. The measured Allan variance of the modulated signal does not converge with increase of number of the data n when $\omega\tau = m\pi$ (m = 1, 2, 3, . . .), but is determined only by the starting time of the measurement. In this case the Eq. (3) indicates the average of the result of measurements. As the result of the measurements by the ARPS agree with that of the calculation by Eq. (3), it has been confirmed that the ARPS functions correctly.

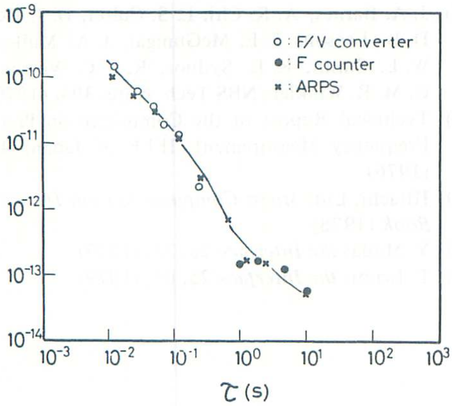

Fig. 8. The result of the frequency stability of He-Xe lasers at 3.51 μm.

Figure 8 shows the result of the measurement of the frequency stability of the laser by using the ARPS and using the conventional measurement system simultaneously. The experimental set up is shown in Fig. 2, and the two lasers are stabilized frequency-offset-locked 3.51 μm He-Xe lasers.[7] The beat frequency was locked to be 2 MHz, and the Allan variance was measured by the ARPS and by the conventional system. To measure the beat frequency for this purpose, authors conventionally used a commercial frequency counter and frequency to voltage (F/V) converter, because the commercial frequency counter yields some errors at the short term shorter than 1 s which comes from the dead time of counting for reset and display.† The F/V converter, however, is less precise for the purpose of measuring the long term stability of authors' lasers.†† Then no reliable Allan variance has been measured in the region between $0.1\,\text{s} < \tau < 1\,\text{s}$. The acquired data of the frequency have been recorded on paper tape and analogue data recorder, respectively, and processed to calculate the Allan variance by an off-line minicomputer (LSI-11). On the other hand, the Allan variance can be measured by the ARPS in wide range of τ, that is, $1\,\text{ms} \leq \tau < 2^{32} \times f^{-1}\,\text{s}$ where f is the frequency of the input signal measured in Hz. The maximum value of the τ depends on accumulation of the counter data using 4 byte length. As the result of the measurement by the ARPS agree with that of the conventional system, it has been confirmed that the ARPS can be used for the real-time measurement of the stability of the laser frequency.

6. Summary

Performances of the ARPS are summarized as follows:

i) The ARPS can be used for real-time measurement.
ii) Real Allan variance is computed without dead time of the gate.
iii) Allan variances of two independent signal sources are available.
iv) The Allan variance in wide range of $1\,\text{ms} \leq \tau < 2^{32} \times f^{-1}\,\text{s}$ is available.
v) More accurate Allan variance is achievable compared with conventional F/V converters, because the ARPS adopts 24 bit frequency counters and accumulate counts on memory of 32 bit length.
vi) The present system is easily handled and with low cost as it is designed for a single purpose system.

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†: The dead time is longer than 50 ms in commercially available frequency counters.

††: σ_y is about 10^{-11} at $\tau = 10^{-1}$ s, in the case of the 3.51 μm He-Xe laser of the authors' group.

As the input stage of the ARPS is composed of simple TTL frequency counters, the ARPS would give us wide variety of application such as the stability measurement of quartz oscillators, and signal frequencies in general. Furthermore, if a V/F converter is used with the ARPS, it can be also used to measure the amplitude stability of any analog signals.

Acknowledgements

The authors are much indebted to E. Hatakoshi of their laboratory for his help to prepare some of electronic circuits in the ARPS. This work is partially supported by Grant-in-Aid for Scientific Research from Ministry of Education, Science and Culture.

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